Machine learning for single particles

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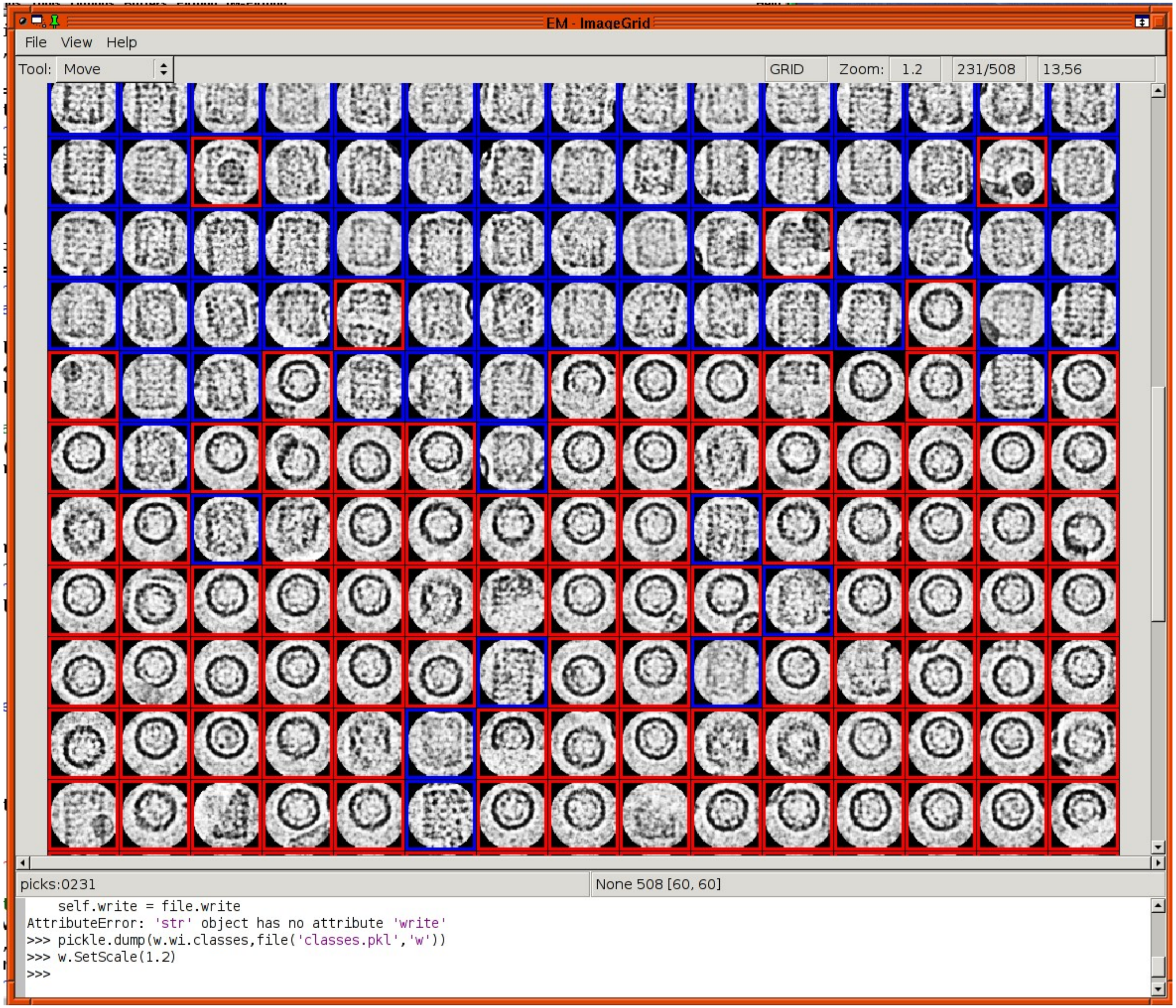
Particle picking

- Problem:
  - Views are randomly distributed on images
  - Must pick regions with particles from image
- Difficulty: high noise ➜ simple template matching does not work
- Approach:

  Initial picks by linear correlation

  Use a Support Vector Machine (SVM) to select for correct particles according to a manually chosen data set

Picking by template matching



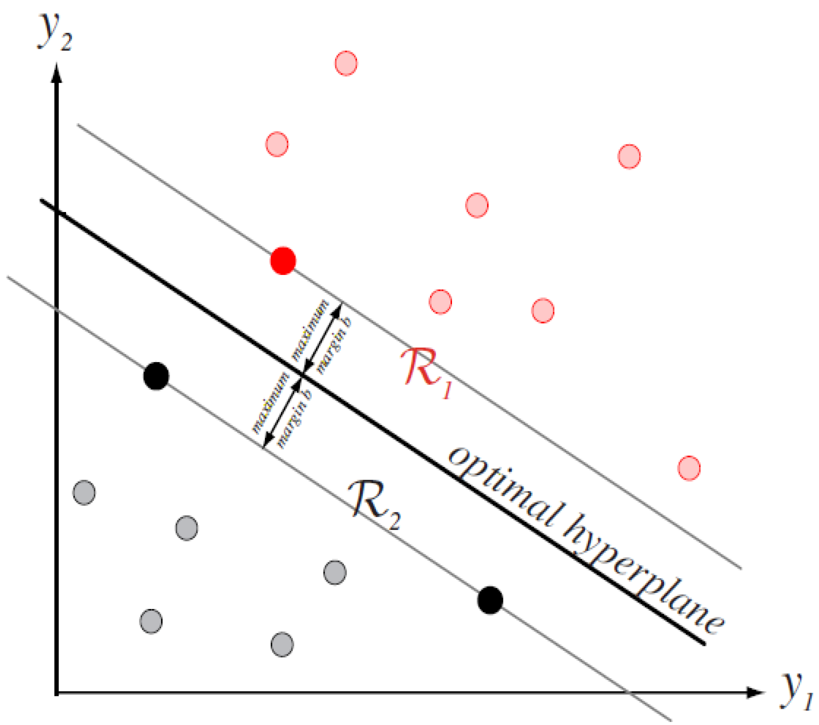
Picking by linear correlation

many mis-picks

Apply SVM to pixel vector (reduced) of the images

Coloring: training data set

Support Vector Machines



From Duda *et al.*, *Pattern Classification*

Machine Learning:

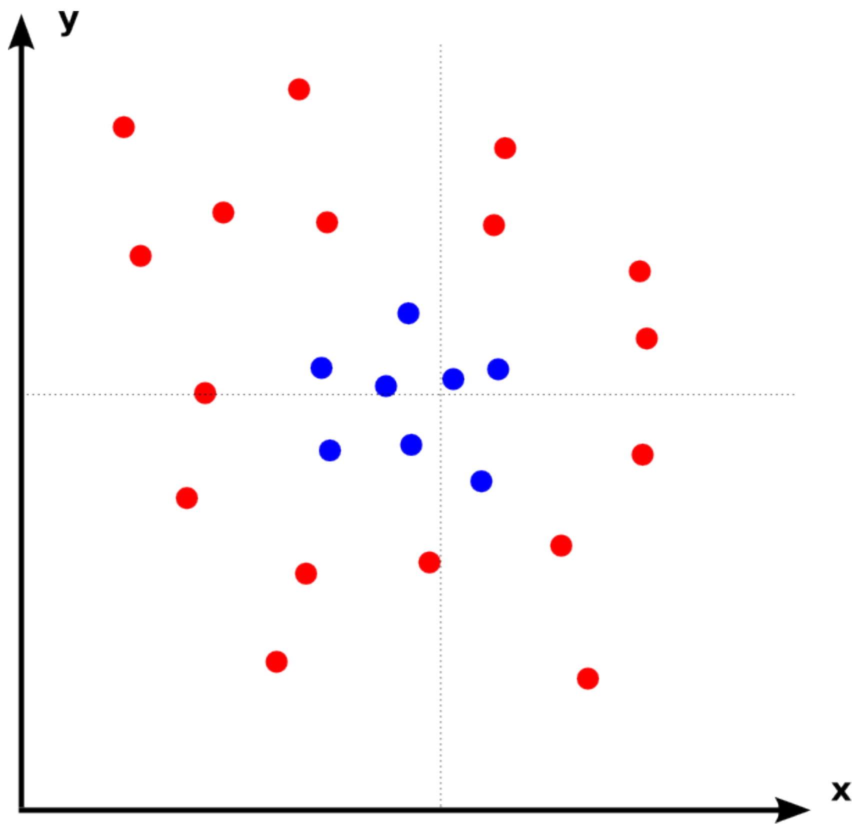
- Training (vs rules)

Support Vector Machine:

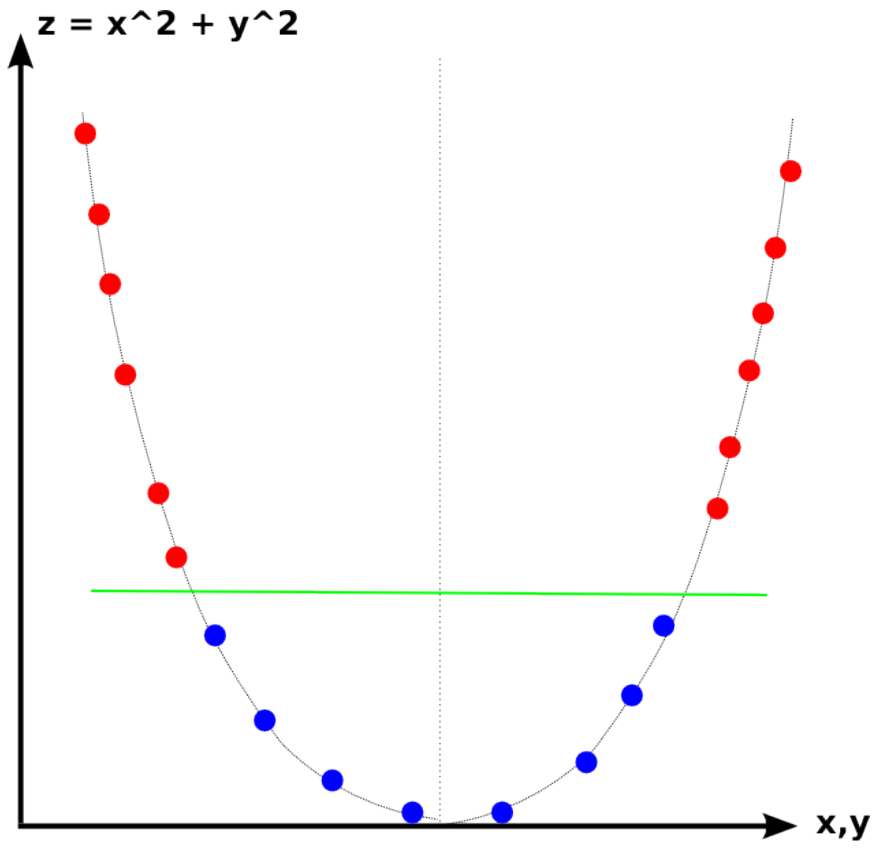
- Linear classifier
- Extended to higher polynomials
- Efficient calculation of the separating hyperplane by duality transform

Non-linearity

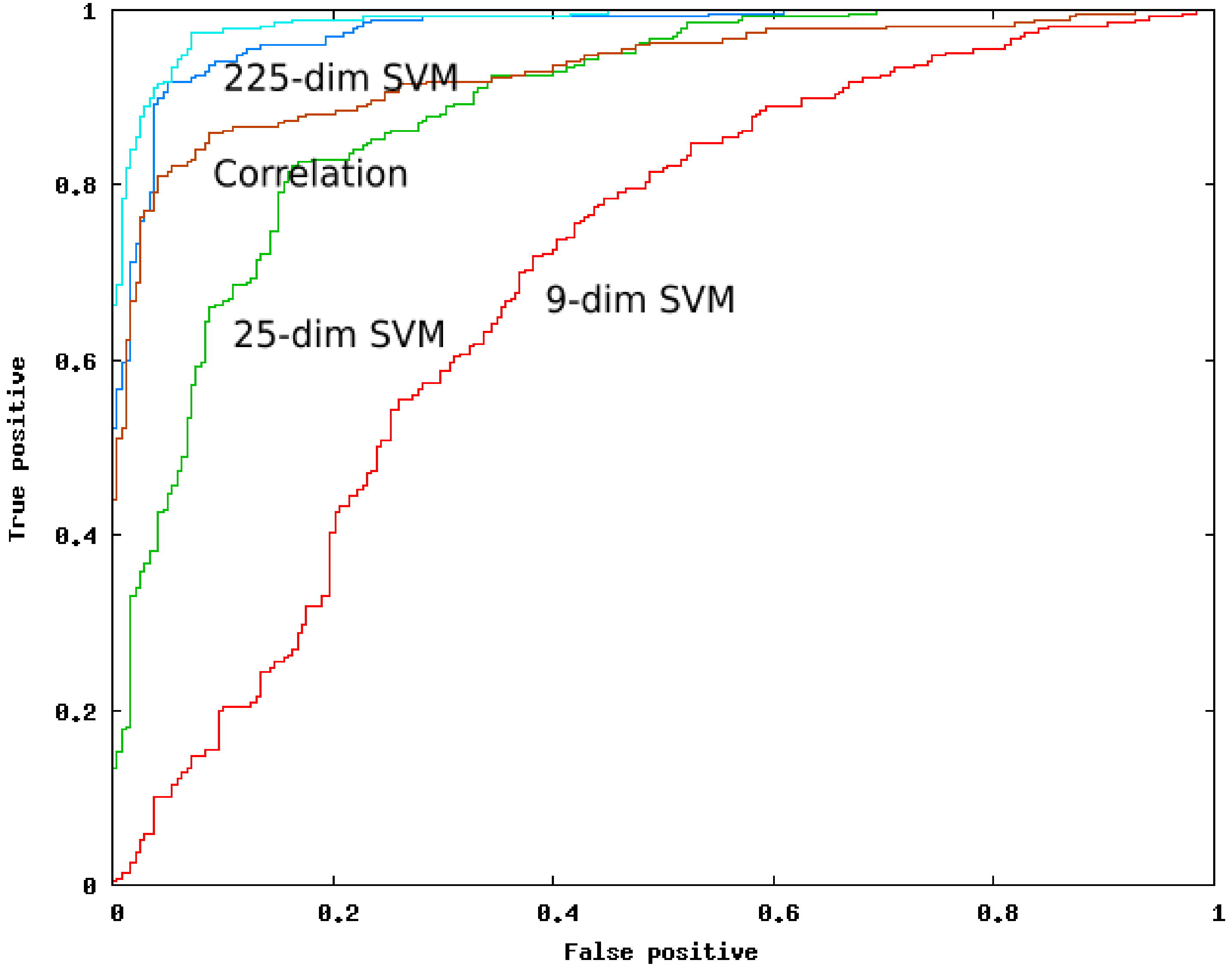
Linearly inseperable



Linearly separable
after introduction of pseudo-variable



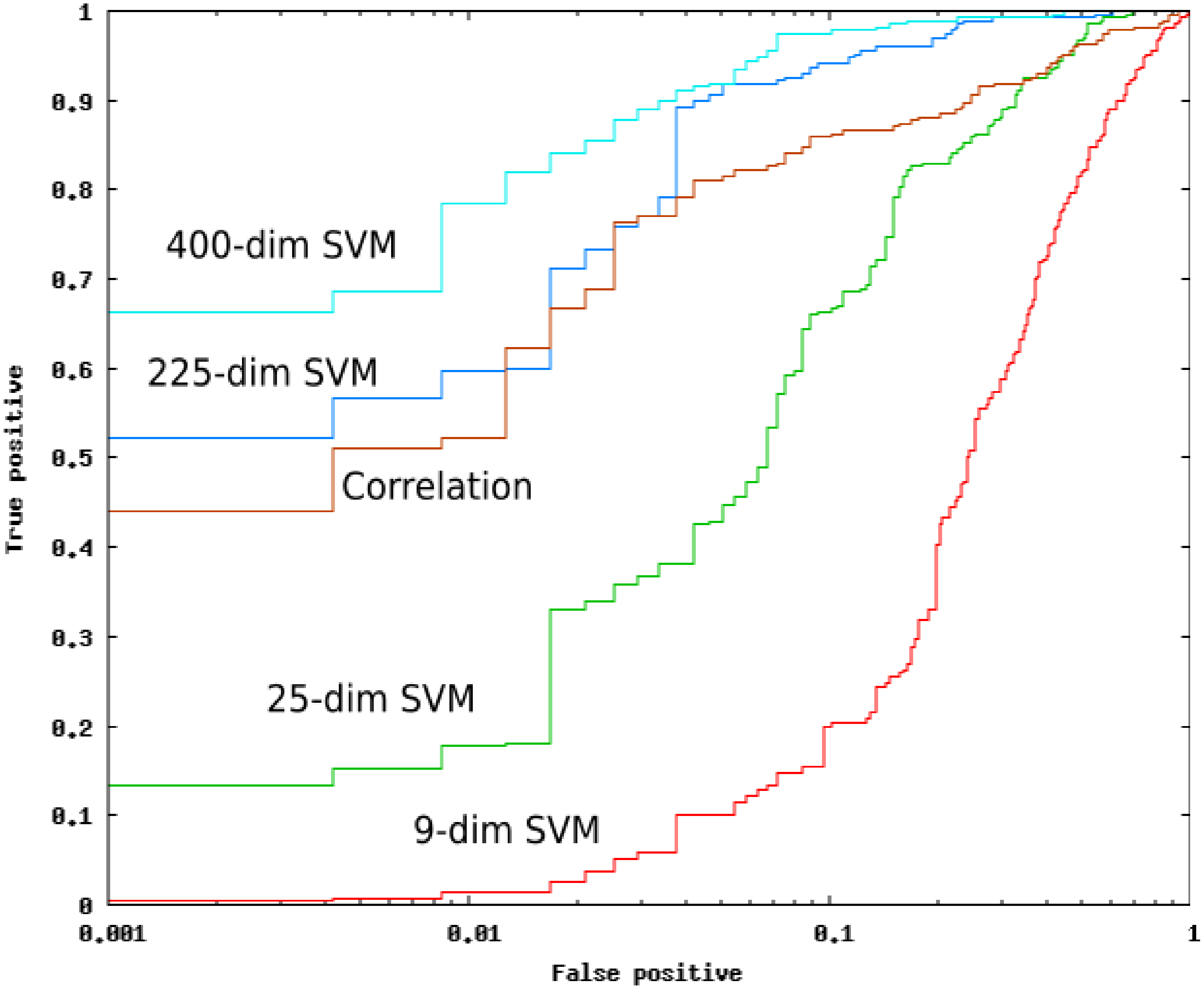
Improving picking using SVMs



Receiver operating characteristics for different feature set sizes

M. Tacke, C. Best 2006

Improving picking using SVMs

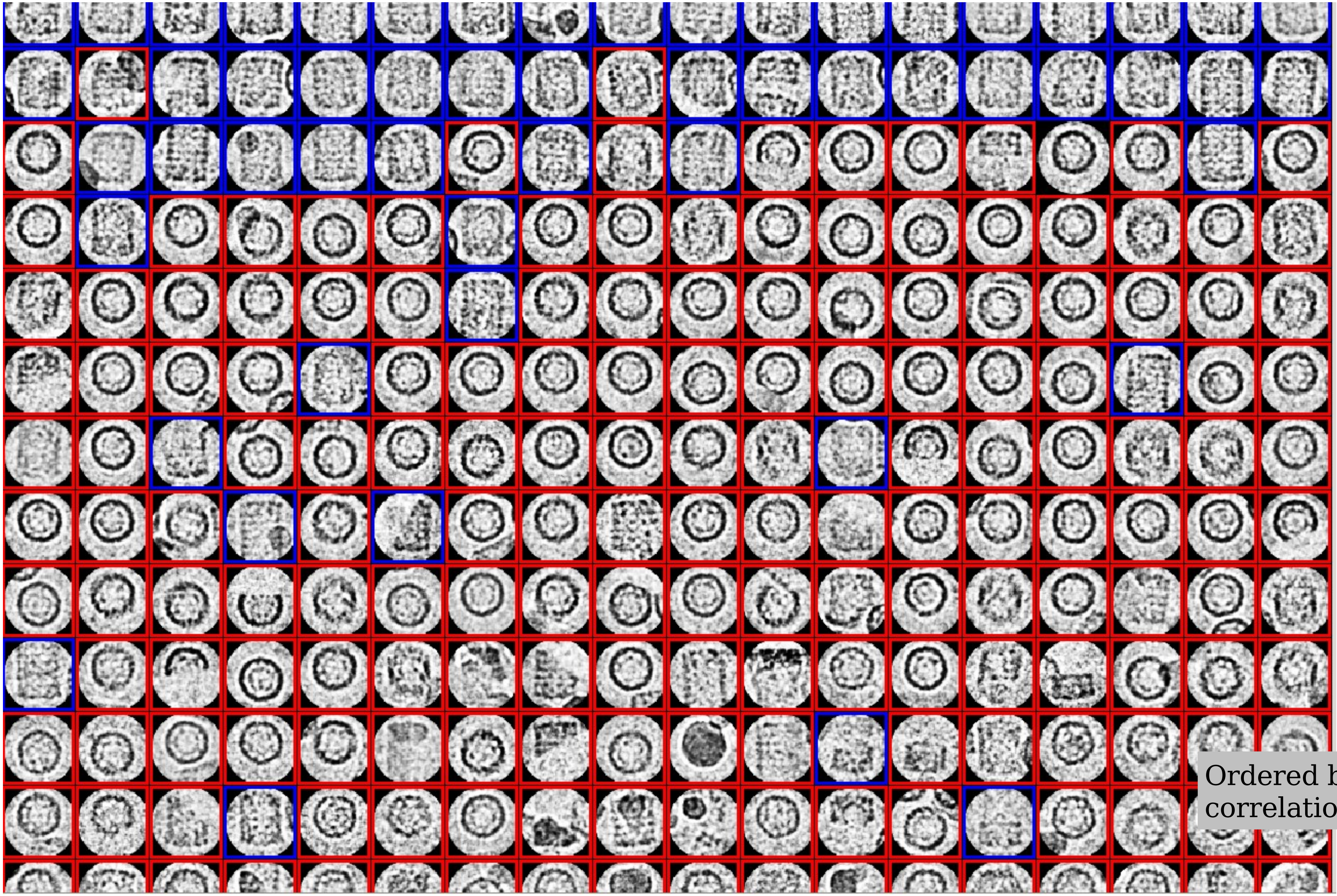


Receiver operating characteristics for different feature set sizes

[Logarithmic scale]

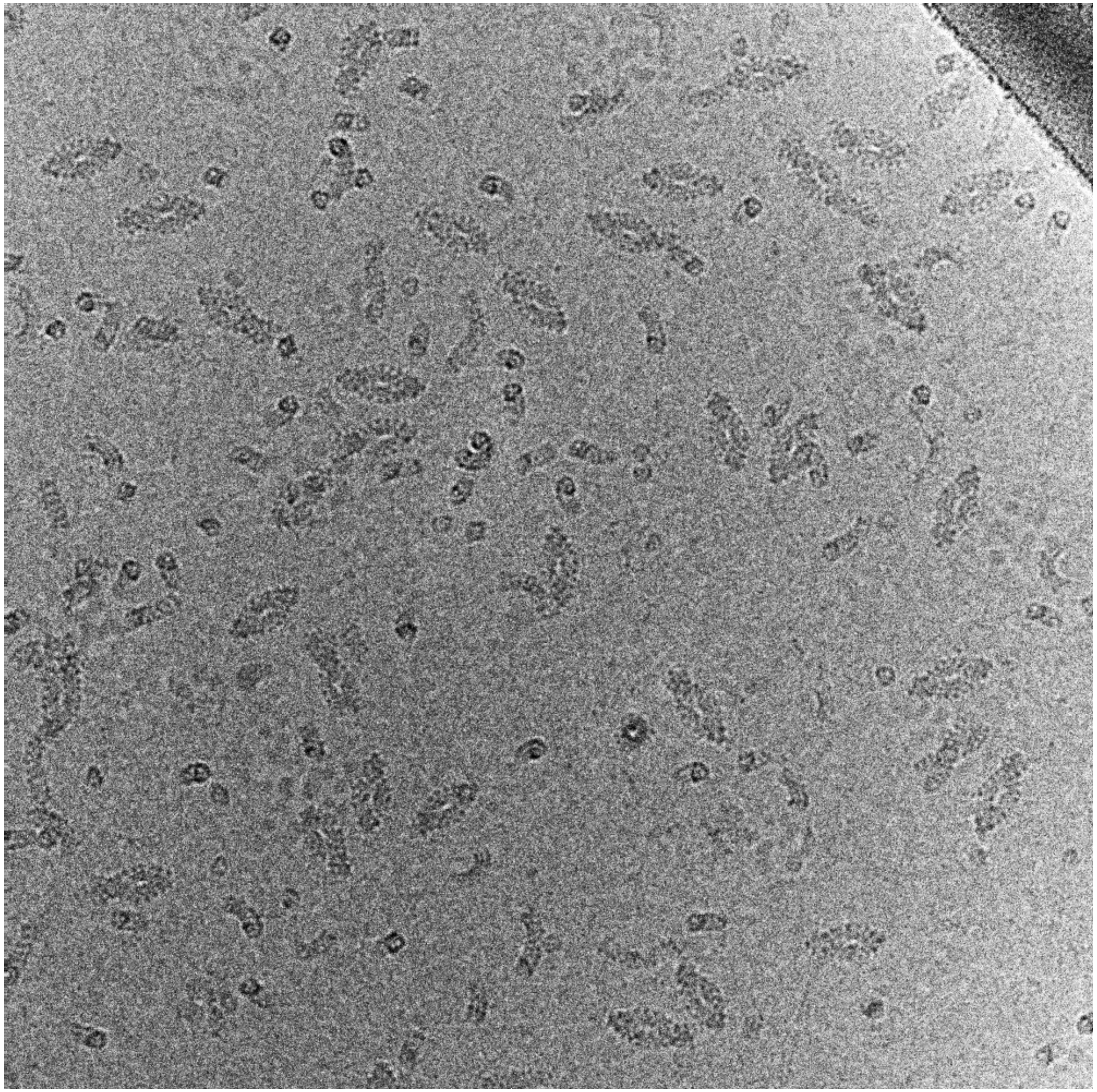
M. Tacke, C. Best 2006

Picking result



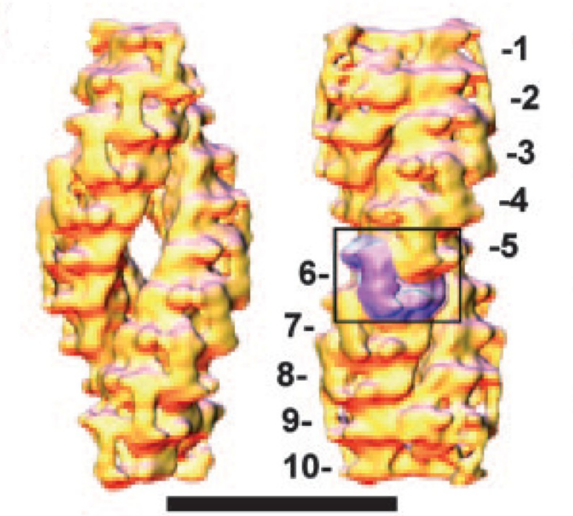
Ordered by correlation value

Sorting views into angles



Tripeptidyl-peptidase II
(TPP II)

courtesy of B. Rockel, Martinsried



Sorting views into angles

- Problem:

  How can we sort the views of a particle according to the viewing angle (elevation, azimuth) ?

- Answer:

  Similar angles ➜ similar images

- Does not require any knowledge about the actual 3D model!
- HOW?

  Parameter estimation in a probabilistic model

What are Bayesian models?

- Probability distribution ↔ Belief about reality
- What do we know?

Images that correspond to nearby viewing angles should be similar

- This can be expressed by a probability distribution

$$P(M|\phi)$$

"Belief that images M are compatible with the angular assignments ϕ"

- Building block:

$$P(M|\phi; M_{0,} \phi_0)$$

"Belief that two images are compatible with each other given their angular assignment"

Model-free particle classification

- Probabilistic model:

$$P(M|\phi; M_0, \phi_0) = \left(\frac{1}{2\pi\kappa(|\phi-\phi_0|)}\right)^{D/2} \exp\left(-\frac{|M-M_0|^2}{2\kappa(|\phi-\phi_0|)^2}\right)$$

Angular distance-to-similarity kernel

- Probability for an image M given an assigned angle ϕ, a reference image $M^{(0)}$, and a reference angle $\phi^{(0)}$:

Gaussian with a width that gets wider when the images are farther apart.

Parameter estimation

- Problem:

We **do** know the images – why would we care about their probability distribution?

- Bayesian parameter estimation:

Images

$$\mathrm{P}(M|\phi) \Leftrightarrow \mathrm{P}(\phi|M)$$

Viewing angles

- This is done using **Bayes' formula**
- Simplified version: **Maximum-likelihood estimation**

$$\phi = \max \mathrm{P}(M|\phi)$$

The best angular assignments are those which make the images most probable

Self-organizing point map

- Joint probability distribution:

$$P(\{M^{(n)}\}|\{\phi^{(n)}\}) = \prod_{i=1}^{N} P(M^{(i)}|\{M^{(i')}, \phi^{(i')}\})$$

- Maximum-likelihood principle ➜ Hamiltonian:

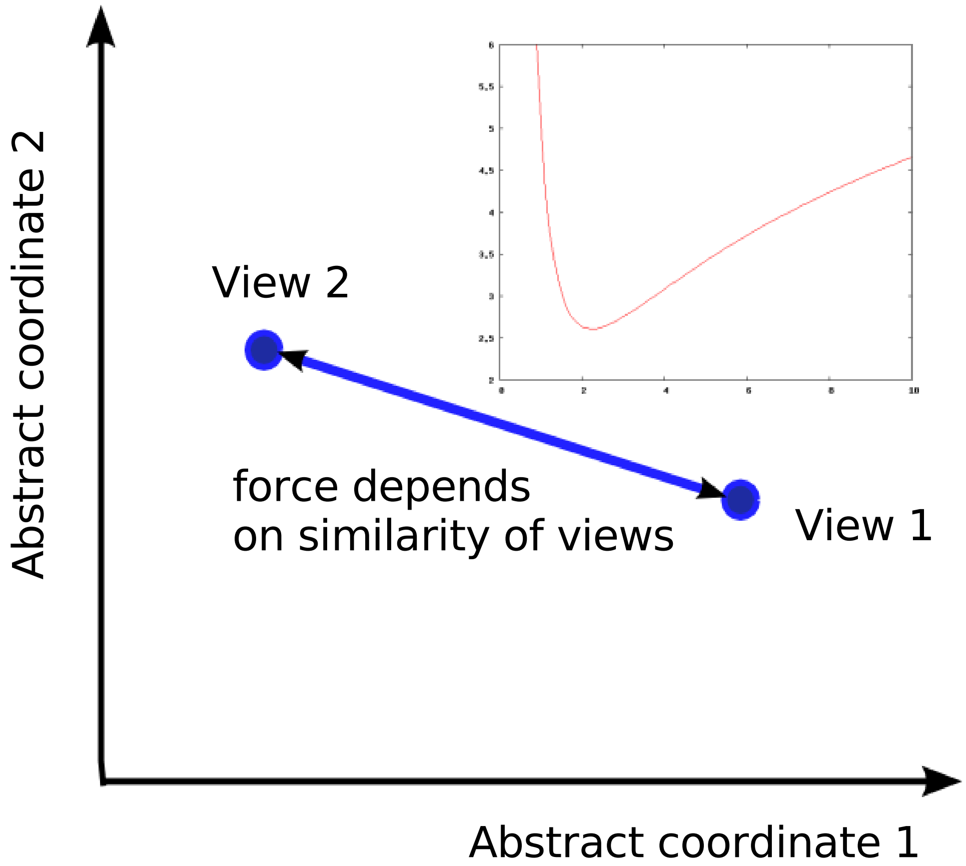
$$-\ln L(\phi) = \sum_{n,m}\left(\frac{D}{2}\ln 2\pi\kappa(|\phi^{(n)}-\phi^{(m)}|) + \frac{|M^{(n)}-M^{(m)}|^2}{2\kappa(|\phi^{(n)}-\phi^{(m)}|)^2}\right)$$

Attractive force   Repulsive force

Point-to-point potential ➜ multidimensional scaling

- Gradient descent solution

Optimization process

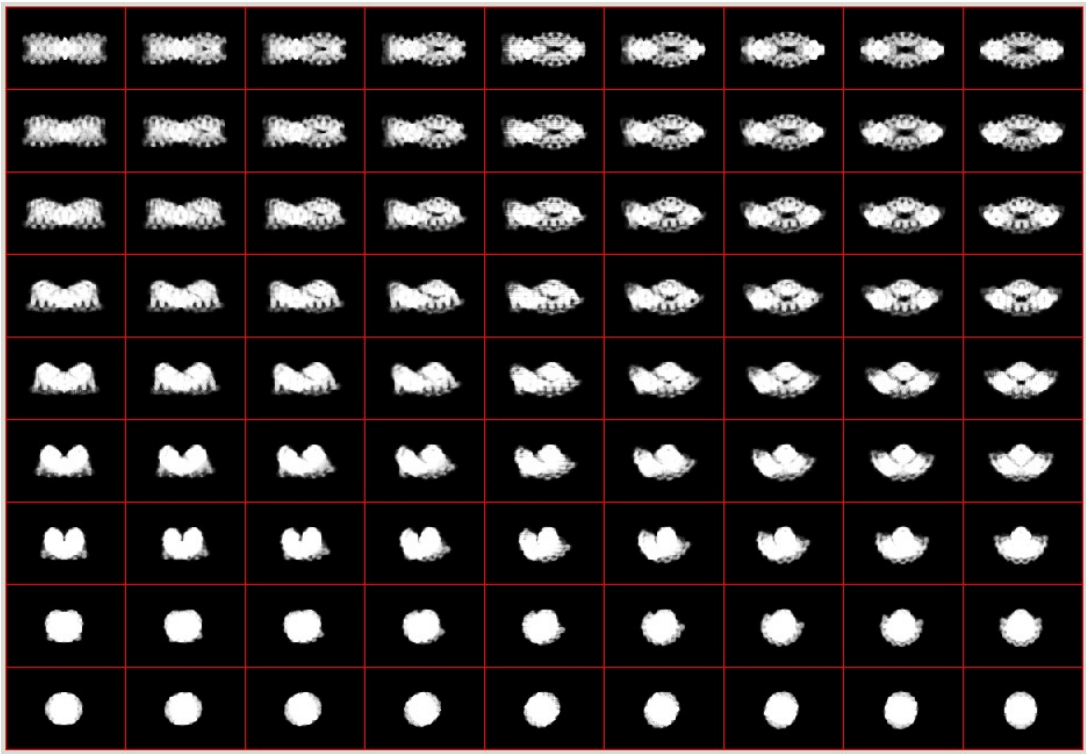


"Spring embedding"

Attractive and repulsive "force" between points (=images)

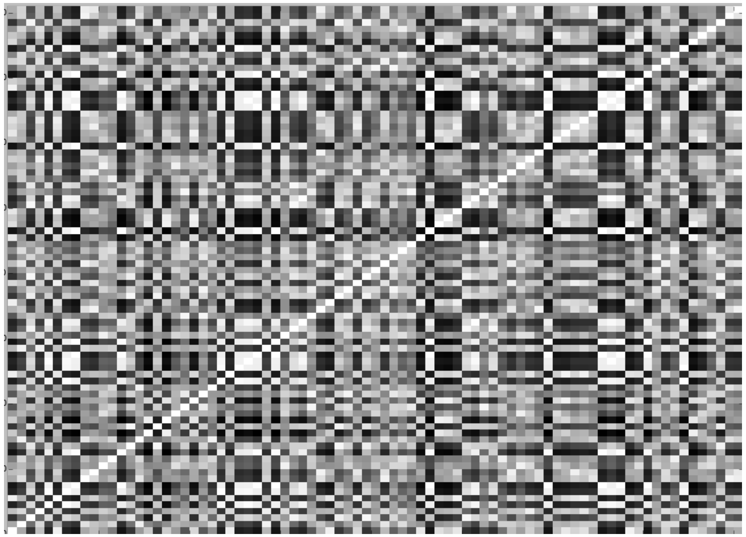
Minimum (=optimum) is determined by how similar the images appear

Similarity matrix



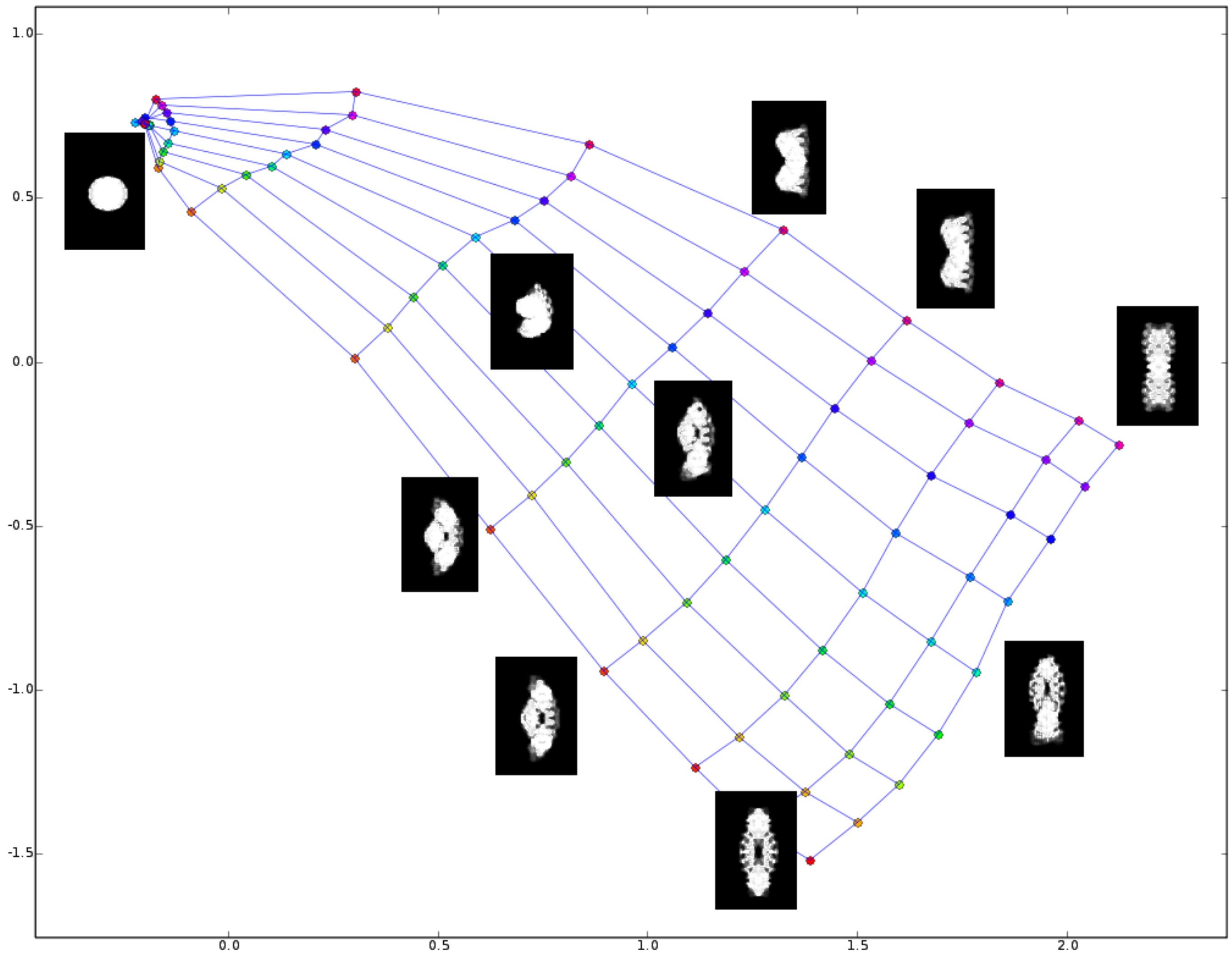
9x9 projections
of TPP2

Correlation matrix:



Pairwise correlation
max. over translations and
rotations

Result



Good representation of original distribution of viewing angles

Good as an initial model for iterative refinement

Tomographic classification